Building a Foolproof Navigation System: Fuzzy Logic Emulating the Brain

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Abstract:

Our aim is to help build a machine that can reduce the possibility of mishaps in navigation to zero. For that devise a new system of numbers, in which the real numbers are represented on the y-axis and complex numbers on the x-axis. Inside such a system, we incorporate the equivalent Ideal Fuzzy Logic that can be used by the machine to predict and avoid mishaps.

Introduction:

Natural processes are vastly emergent phenomena and each new result is always the source of new emergence. To cope with nonlinear control problems, binary logic is no longer sufficient. What we need is an ideal Fuzzy Logic that not only can process complex numbers with utmost efficiency, but also 'thinks' in terms of complex numbers. Such a system also needs to be as simple as possible for us and for machines to work with.

A perfectly efficient navigation system will be able to receive inputs that have all sets of possible values. These values may be real or imaginary. Such a system will be particularly efficient in dealing with imaginary numbers and will be able to reduce the probability of a mishap to zero.

The Fuzzy logic that we have today is yet to incorporate imaginary numbers with the desired level of satisfaction, and many serious problems remain in this.

We develop a new system of numbers that can make use of the equivalent Fuzzy Logic inside it. A machine that runs on such a system will be able to predict and avert mishaps completely.

Let us consider a system of numbers, where all complex numbers are real and all real numbers are complex, i.e.,

$$R \Leftrightarrow C \qquad \text{...(1)},$$

R and C being real numbers and complex numbers, respectively.

In such a system, all individual positive numbers are negative and all individual negative numbers are positive, for complex numbers, while all original real numbers are multiplied by i.

Thus,

$$C=x+iy \Rightarrow C'=-x+\sqrt{1}(-y) = -(x\pm y) \in R \qquad \text{…(2.1)}$$

And

$$R \Rightarrow R'=-iR =-(0+iR) \in C \qquad \text{…(2.2).}$$

Thus, in such a system, we have two lines of numbers:

1. The set of originally complex numbers, transformed into real ones, represented on the x-axis.
2. The set of originally real numbers, transformed into complex ones, represented on the y- axis.

The original system transforms, therefore, into two separate and mutually perpendicular number lines, with individual numbers being points on the corresponding lines.

An Ideal Navigation System:

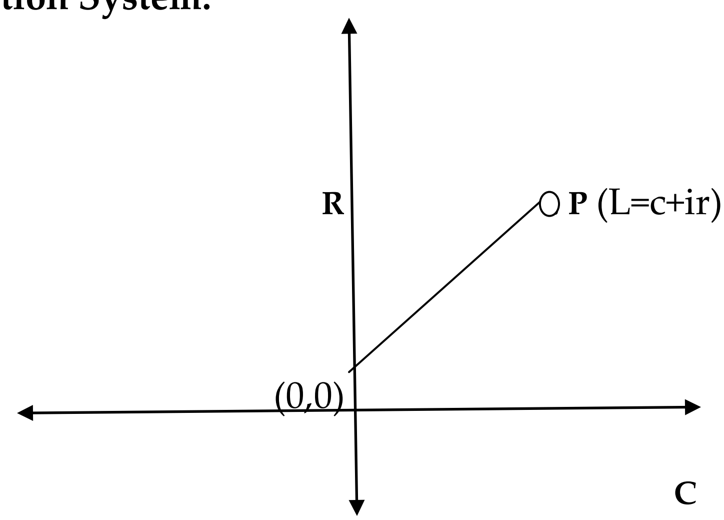

Fig. 1: A simple illustration of signal-processing by a machine running on the new system.

Fig. 1 shows a simple illustration of signal processing by a machine. The set of complex numbers on the x-axis and the set of real numbers on the y-axis give rise to the parameter L for any given point P in the super-imaginary complex plane of decision making. The parameter L represents both the real and the imaginary parts of the signal and incorporates them into the abstract decision making/perception plane.

In the complex decision/perception plane:

$$L=c+ir \qquad \ldots(3).$$

c and r being the complex and real coordinates for the point P.

The basic difference between *classical se*ts and *fuzzy sets* is that while classical sets allow only a dual degree of membership, a fuzzy set can incorporate any real value between the dual states concerned. A characteristic membership function assigns 0 to an element that is not a member of a given classical set, while it assigns a value of 1 to an element that is a member of that set. The degree of membership to a fuzzy set can take any value in the real unit interval [0, 1].

In our decision/perception plane a fuzzy set $\mathrm{L_F}$ may be defined as:

$$\mathrm{L_F}: \mathrm{L} \rightarrow [0,1], \qquad \ldots(4).$$

where L is a domain of elements (universe of discourse).

For every particular value of a variable $\mathrm{L_i} \in \mathrm{L}$ the degree of membership to fuzzy set $\mathrm{L_F}$ is $\mathrm{L_F}\ (\mathrm{L_i})$.

Equation (4) describes how we can incorporate a fuzzy complex number or FCN in our decision/perception plane.

$\mathrm{L_F}$ in the universe of discourse L is defined by the complex membership grade function $\mu\mathrm{L_F}(\mathrm{L_i})$. The complex membership grade function or CMG is defined as:

$$\mu\mathrm{L_F}(\mathrm{L_i}) = \mathrm{L_F}(\mathrm{L_i})e^{ic} \qquad \ldots(5).$$

The Cartesian representation of CMG for $\mu L_F(L_i) = \mu L_F(c_i + ir_i)$ is:

$$\mu(c_i, r_i) = \mu(c_i) + ir_i \qquad \text{…(6)}$$

And, the polar representation is:

$$c_i e^{isr} \qquad \text{…(7),}$$

the scaling factor s being in the interval $(0, 2\pi]$.

The degree of fulfillment or DOF of any given proposition follows CMG and lies in the interval [0,1].

According to the definition of transformation of coordinates:

$$\mu(c_i, r_i) \Leftrightarrow c_i e^{isr}$$

The operators $\wedge$ and $\vee$ defining t-norm and s-norm respectively and L_i being the set of fuzzy numbers concerned, the fuzzy set of a function of L_i has the membership function:

$$\mu\left(c_i^{'}, r_i^{'}\right) = \vee_{c_i^{'} = f(L_i)}[\mu(c_1, r_1) \wedge \mu(c_2, r_2) \wedge \mu(c_3, r_3) \ldots \wedge \mu(c_n, r_n)] \qquad \text{…(8).}$$

Operators and Functions in the New Logic:

OR:

The maximum s-norm (smax) may be used to calculate the DOF concerned. P and Q being two possible values under consideration, DOF(L= $P+Q$)= $smax[DOF(P), DOF(Q)] = \max[DOF(P), DOF(Q)]$ …(9).

Truth Table

P	Q	max(L)=max(P,Q)
0	0	0
0	1	1
1	0	1
1	1	1

AND:

The minimum t-norm (tmin) may be used to calculate the DOF concerned. P and Q being two Possible values under consideration,

DOF(L= $P \cdot Q$)= $tmin[DOF(P), DOF(Q)] = \min[DOF(P), DOF(Q)]$ …(10).

Truth Table

P	Q	min(L)=min(P,Q)
0	0	0
0	1	0
1	0	0
1	1	1

NOT:

The complement or negation is used to calculate the DOF concerned. $\bar{L}$ being the complement of L,

$$\mathrm{DOF}(\bar{L}) = 1\text{-}\mathrm{DOF}(L) \qquad \ldots(11).$$

Truth Table

L	$\bar{\mathbf{L}}$
0	**1**
1	**0**

De Morgan's Operations:

De Morgan's Involution holds as:

$$\neg(P \wedge Q) = \neg P \vee \neg Q$$

and

$$\neg\neg L = L \qquad \ldots(12).$$

where $\neg$ is the NOT operator.

Also, De Morgan's laws hold as:

NOT(P AND Q) ⇔(NOT P) OR (NOT Q)

And

NOT (P OR Q) ⇔(NOT P) AND (NOT Q) …(13).

Now, the relations (13) also translate to:

$$1 - \min[DOF(P), DOF(Q)] \Leftrightarrow \max[1 - DOF(P), 1 - DOF(Q)]$$

And

$$1 - \max[DOF(P), DOF(Q)] \Leftrightarrow \min[1 - DOF(P), 1 - DOF(Q)] \qquad \ldots(14).$$

Chaotic Fuzziness:

In order to incorporate chaotic instances, the ideal Fuzzy Logic may start with a basic starting measure as the standard reference, from which it will calculate the required differences with respect to other values. It may even take one attractor as its reference. However, the manners of functionality of the fuzzy operations that both will follow will be just the same.

For a given transport of parameterization of the degree of membership between an initial and a final point in consideration, let the trajectory of the initial point of reference $L_o = L(o)$ be denoted by,

$$L(t) = f^t(L_o)$$

Expanding $f^t(L_o + \delta L_o)$ to linear order, the evolution of the distance to a neighbouring trajectory $L_i(t) + \delta L_i(t)$ is given by the Jacobian matrix J,

$$\delta L_i(t) = \sum_{j=1}^{d} J^t(L_o)_{ij} \ \delta L_{oj} \, ,$$

$$J^t(L_o)_{ij} = \frac{\boldsymbol{\delta L_i(t)}}{\boldsymbol{\delta L_{oj}}} \qquad \text{…(15).}$$

A trajectory of the shift in degree of membership as moving on the decision/perception plane is specified by two position coordinates and the direction of motion. The Jacobian matrix describes the deformation of an infinitesimal neighbourhood of $L(t)$ along the shift.

Holding the hyperbolicity assumption (i.e., for large n the prefactors a_i, reflecting the overall size of the system, are overwhelmed by the exponential growth of the unstable eigenvalues Λ_i, and may thus be neglected), to be justified, we may replace the magnitude of the area of the ith strip $|B_i|$ by $\frac{1}{|\Lambda_i|}$ and consider the sum,

$$\lceil n = \sum_{i}^{n} \frac{1}{|\Lambda_i|};$$

Where the sum goes over all periodic points of period n. We now define a generating function for sums over all periodic orbits of all lengths,

$$\lceil z = \sum_{n=1}^{\infty} \lceil n\, z^n \qquad \text{…(16).}$$

For large n, the nth level sum tends to the limit $\lceil n \rightarrow e^{-n\gamma}$, so the escape rate γ is determined by the smallest $z = e^{\gamma}$ for which $\lceil z$ diverges,

$$\lceil z \approx \sum_{n=1}^{\infty} (ze^{-\gamma})^n = \frac{ze^{-\gamma}}{1-ze^{-\gamma}} \qquad \text{…(17).}$$

Making an analogy to the Riemann zeta-function, for periodic orbit cycles,

$$\lceil z = -z \frac{d}{dx} \sum_p \ln(1 - t_p);$$

$\lceil(z)$ is a logarithmic derivative of the infinite product

$$\frac{1}{\zeta(z)} = \prod_p (1 - t_p), t_p = \frac{z^{n_p}}{|\Lambda_p|}$$

This represents the dynamical zeta function for the escape rate of the trajectories of quantum-transport. The fraction of initial x whose trajectories remain within B at time t may decay exponentially,

$$\lceil t = \frac{\int_s dx\ dy\ \delta[y - f^t(x)]}{\int_s dx} \rightarrow e^{-\gamma t} \qquad \ldots(18).$$

Considering a collection of such points and applying a statistical approach, the logistic equation (due to May, 1967) for L can be written as,

$$L_{t+1} = KL_t[1 - L_t] \qquad \ldots(19).$$

where K is a constant.

Also, the quadratic map (due to Lorentz, 1987) can be written as:

$$L_{t+1} = K - (L)_t^2 \qquad \ldots(20).$$

All trajectories described by the quadratic map become asymptotic to $-\infty$ for $K < -0.25$ and $K > 2$.

As we deal with the flow of a given measure towards a given reference, the expression for the attractor for each such point can be written as,

$$L^* = \left(1 - \frac{1}{K}\right) \qquad \ldots(21).$$

where $O < K < 4$.

L^* is a point in the desired dimensional plot into which the trajectories seem to crowd. As we do not need to deal with more than one attractor or periodic point, the trajectories will tend to revisit only the attractor point concerned, to the desired level of accuracy of observations and calculations.

In equation (21), for $K \geq 3$, the trajectory behaviour becomes increasingly sensitive to the value of K. There are a few more points to be noted regarding the dependence of the trajectory behaviour on the values of K:

1. For $K \leq 1$, the attractor is a fixed point and has a value 0.
2. For $1 < K < 3$, the attractor is a fixed point and its value is > 0 but < 0.667.

3. For $3 \leq K \leq 3.57$, period doubling occurs, with the attractor consisting of $2, 4, 8$, etc., periodic points as K increases within that range.
4. For $3.57 < K \leq 4$, we have the region of chaos, where the attractor can be erratic (chaotic with infinitely many points) or stable.

For all calculations, the desired conditions may be placed at the attractor. A trajectory never gets completely and exactly all the way into an attractor though, but only approaches it asymptotically. In the region of chaos, we apply the method of searching for windows or zones of K-values for which iterations from any initial conditions will produce the periodic attractor, instead of a chaotic one. For the logistic equation, the most common such zone lies at $K \approx 3.83$ and for the quadratic map, at $K \approx 1.76$.

Using Lyapunov exponents for the measure L, and replacing $2c\left(\frac{\lambda}{D}\right)$ by a quantity $'\tau'$, we have:

$$\frac{d}{d\tau} f^{n}(L) = \frac{\delta n}{\delta o}$$

i.e.,

$$\frac{\delta n}{\delta o} = \prod_{i=1}^{n} f^{'}(L_i) \qquad \ldots(22).$$

$$b = \frac{1}{n} \log_e \left(\frac{\delta n}{\delta o}\right)$$

i.e.,

$$b = \frac{1}{n} \sum_{i=1}^{n-1} \log_e \left| f^{'}(L_i) \right| \qquad \ldots(23).$$

where b is a constant (the local slope of all possible measures), and

$$\Psi = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \log_e \left| f'(L_i) \right| \quad \ldots(24).$$

where Ψ is a constant.

Let $L_t = c_t + ir_t$ and $L_{t+1} = c_{t+1} + ir_{t+1}$ be corresponding FCN measures with complex membership grade function or CMG as $\mu L_F(L_t)$ and $\mu L_F(L_{t+1})$, respectively. We may then perform the basic arithmetic operations as:

Addition:

$$L_t + L_{t+1} = (c_t + c_{t+1}) + i(r_t + r_{t+1})$$

The corresponding membership function is:

$$\mu L_F(L_t + L_{t+1}) = \vee_{L_t + L_{t+1}} [\mu L_F(L_t) \wedge \mu L_F(L_{t+1})]$$

Subtraction:

$$L_t - L_{t+1} = (c_t - c_{t+1}) + i(r_t - r_{t+1})$$

The corresponding membership function is:

$$\mu L_F(L_t - L_{t+1}) = \vee_{L_t - L_{t+1}} [\mu L_F(L_t) \wedge \mu L_F(L_{t+1})]$$

Normalization of L_i:

Considering n number of measures, we have the normalized measure for L_i as:

$$Nor(L_i) = \frac{L_i - \min(L_i)}{\max(L_i) - \min(L_i)} \qquad \ldots(25).$$

$D(P,Q)$ being the distance measure between two normalized fuzzy sets P and Q, within the measure L_i, the degree of match between them is denoted by:

$$M(P,Q) = 1 - D(P,Q)$$

If, $p \in P$ and $q \in Q$, then the maximum distance between the nearest points in P and Q is the Hausdorff distance between P and Q:

$$H(P,Q) = max_{p \in P}[\min_{q \in Q} D(P,Q)] =$$
$$max[\sup_{p \in P} \inf_{q \in Q} D(P,Q), \sup_{q \in Q} \inf_{p \in P} D(P,Q)],$$

where sup represents the supremum and inf the infimum.

IF-THEN relations may be evaluated using corresponding DOFs. As such relations are of fundamental importance in any logical construct, they are of interest to us here. We may use a weighted scaling measure S to get the DOF of the final result of a given IF-THEN relation. For this, we break the IF-THEN relation into its constituent parts i.e., the condition part (IF) and the result part (THEN). As the IF relation is always of the form *constituent 1 AND constituent 2*, we may write the condition DOF as:

$$DOF_{Condition} = \min[DOF(Constituent\ 1), DOF(Constituent\ 2)]$$

The product of the scaling measure S and $DOF_{Condition}$ gives the final result as:

$$DOF_{Result} = S(DOF_{Condition}) \qquad \ldots(26).$$

The scaling measure S may be taken as the corresponding Hausdorff distance between the constituents.

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